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# Staircase Wear Simulation and Nondestructive Assessment Based on an Optimized Archard Model

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**Abstract.** Staircase wear constitutes critical archaeological evidence for reconstructing usage patterns and occupation dynamics. This study introduces a Staircase Wear Model for simulating the formation and evolution of tread wear and a complementary Stair Wear Assessment Model for non-destructive evaluation across diverse contexts. An Optimized Archard framework relates stepwise material loss to use frequency through calibrated contact mechanics, accommodating heterogeneous substrates and surface conditions. Directional asymmetries are addressed by a Traveling Direction Judgment Model that distinguishes ascent and descent trajectories and maps their respective load paths onto tread geometry. Crowd effects are incorporated via an improved strategy that infers concurrent user groups, estimates effective capacity, and delineates same-wear regions reflecting spatial interactions among footprints. The integrated workflow yields temporally plausible wear fields, supports inverse inference of circulation intensity and preferred movement directions, and provides reproducible indicators for comparing staircases within and between sites. Emphasis on interpretability, computational efficiency, and compatibility with photogrammetry or 3D scanning inputs facilitates routine application in archaeological documentation, conservation planning, and hypothesis testing.

**Keywords:** Archard Wear Model, Archaeological Analysis of Staircases, Staircase Usage Patterns.

## 1. Introduction

The accurate prediction of power load is of great significance for the electric power production and the safe operation of the power grid and the national economy [1]. Short term load forecasting is an important part of energy management system. The prediction error directly affects the analysis results of subsequent safety check of power grid, which is of great significance for dynamic state estimation, load scheduling and cost reduction [2-4]. Traditional prediction methods are based on linear regression, such as time series method, analysis method and pattern recognition method has defects of respectively [5].Introduction: Stair wear has emerged as a valuable proxy for reconstructing human movement, maintenance regimes, and occupation intensity in built environments, yet quantitative approaches that fuse surface metrology, wear mechanics, and crowd behavior remain fragmented. Recent advances in non-destructive documentation—especially close-range photogrammetry and terrestrial laser scanning (TLS)—now permit millimetric to sub-millimetric recovery of tread microtopography, enabling spatially explicit maps of polish, rounding, and rill formation on heritage stone or timber stairs, and supporting repeat surveys for change detection. These techniques have been operationalized in field workflows for artifact and architectural traceology, and increasingly for continuous architectural monitoring, establishing a robust data backbone for computational wear analysis.

Methodologically, contact-mechanics models derived from Archard’s law continue to underpin predictive and inverse wear studies across engineering and biomedical domains. Contemporary evaluations emphasize that classical formulations are tractable and broadly applicable but require context-specific calibration of pressure, sliding distance, hardness, and wear coefficients, as well as careful handling of heterogeneous materials and mixed lubrication or abrasive regimes—conditions typical of archaeological stair substrates exposed to weathering and episodic maintenance. Integrating these refinements improves the fidelity of linking cumulative passages to volumetric loss and spatial wear gradients on treads and nosings.

A second thread concerns the directionality and kinematics of stair use. Biomechanics literature documents asymmetric foot placement, foot posture at initial contact, and risk-modulating behaviors during descent versus ascent, implying distinct load paths and contact patches that should be encoded in wear simulators. Empirical studies of stair descent report behavioral and postural features that alter local pressure distributions and contact durations over the tread surface, offering measurable priors for direction-aware wear allocation and for validating inferred traffic directions from preserved wear fields [6].

Third, the intensity of simultaneous use and the resulting lateral dispersion of footsteps are central to translating individual gait mechanics into population-level wear. Crowd modeling has progressed rapidly through agent-based simulation (ABM), synthesizing route choice, speed adaptation, and bottleneck dynamics with validation across evacuation and circulation scenarios; these frameworks provide realistic temporal profiles of occupancy and throughput for staircases in buildings and sites [7].

At the same time, density-based clustering methods have been extended to accommodate multi-density data and to automate parameter selection, making them suitable for segmenting same-wear regions on treads and estimating effective lane counts and capacity from spatial traces or proxy indicators in point clouds or orthomosaics. Despite these advances, there remains a gap in integrated pipelines that begin with high-resolution 3D capture, infer directionally specific contact distributions from biomechanical evidence, propagate these through calibrated Archard-type relations to predict or invert wear, and quantify the role of concurrency via robust clustering on observed or simulated footprints. Existing archaeological applications often focus on documentation without mechanistic inference, while engineering wear models rarely account for crowd heterogeneity, preferential pathing, or the complex surface chemistries and textures of heritage materials.

Addressing this gap requires a unifying, data-efficient framework that (i) maintains interpretability across disciplines, (ii) accepts routine survey inputs from photogrammetry or TLS, (iii) encodes ascent–descent asymmetries in load trajectories, and (iv) links spatially coherent "lanes" of same-wear via density-aware clustering to realistic upper bounds on simultaneous users. The Staircase Wear Model presented here responds to these needs by coupling an Optimized Archard formulation with a Traveling Direction Judgment component and a capacity-oriented clustering module. Together, these elements enable non-destructive assessment of circulation intensity, preferred directions, and maximum concurrent occupancy from observed wear, while also supporting forward simulations for conservation planning and hypothesis testing about past use. The resulting workflow is designed to be transparent, reproducible, and portable across sites with varying materials and documentation standards, thereby advancing stair-focused archaeological inference from qualitative description to calibrated, scenario-ready analysis supported by recent developments in 3D metrology, wear science, biomechanics, and crowd modeling .

## 2. Stair Wear Assessment Model

### 2.1. 2.1 Optimized Archard's Wear Model : Estimations of Use Frequency

To quantitatively characterize the wear of different objects, the Archard wear model is commonly employed. However, to further elucidate the relationship between stair wear and frequency of use, the model propose introducing time as an independent variable into the model. The improved model can be expressed by the following equation:

$$V(n) = \mu \frac{F \cdot d}{H} n \tag{1}$$

where $V(n)$ denotes the degree of wear, here measured in terms of wear volume, $d$ is the displacement at which friction arises. Furthermore, the model subdivide the area of the stair step that meets users' feet into small sections, each measuring 10 mm × 10 mm. Thus, in the subsequent analysis, the overall wear of the staircase can be assessed by examining the data derived from these individual sections. To microscopically capture the wear occurring in these small sections, This model refine equation (1) into the following expression:

$$M_{(x,y)}(n) = \mu \cdot \frac{F_{(x,y)} \cdot d_{(x,y)}}{H} \cdot n_{(x,y)} \tag{2}$$

where $F_{(x,y)}$ denotes the force applied at block $(x,y)$, $d_{(x,y)}$ denotes the number of uses at block $(x,y)$, and $M_{(x,y)}$ denotes the degree of wear at block $(x,y)$, which is still measured in terms of wear volume. Shifting the terms gives the following equation:

$$n_{(x,y)} = \frac{M_{(x,y)}(n) \cdot H}{\mu \cdot F_{(x,y)} \cdot d_{(x,y)}} \tag{3}$$

After processing the data from all the sections, the weighted-average method is applied to determine the total number of staircase uses per unit time

$$n_{total} = \frac{\sum n_{(x,y)} \cdot S_{(x,y)}}{\sum S_{(x,y)}} \tag{4}$$

where $n_{total}$ denotes the total number of times the entire staircase has been used per unit of time, and $S_{(x,y)}$ represents the area of the block $(x,y)$. In the context of this problem, $S_{(x,y)}$ is a constant, specifically 1cm$^2$.Based on this model, The overall frequency of staircase use is then estimated.

## 2.2. Direction Preference Analysis

BP neural network is back propagating, mainly composed of three parts: input layer, middle layer and output layer. The number of nodes in the input and output layers is relatively easy to determine, but the determination of the number of nodes in the hidden layer is a very important and complex problem. In considering whether there is a specific preference for the direction of travel on stairs, it is well established that stair ascent and descent produce distinct plantar-pressure distributions. This variation in pressure leads to distinct wear characteristics for each direction.

According to the references[7-10], In the process of ascending the stairs, the user primarily relies on the forefoot to propel the body upward, with the heel making brief contact and providing less support. As a result, the pressure on the bottom of the foot typically increases gradually from the heel to the front of the foot. In contrast, when descending the stairs, the pressure distribution tends to be more uniform or slightly shifted towards the back of the foot, due to the need to absorb and mitigate the impact of the descent. To visualize this result, the model provide the force diagrams of a person's foot during the ascent and descent of a staircase, as shown in Figure 1.

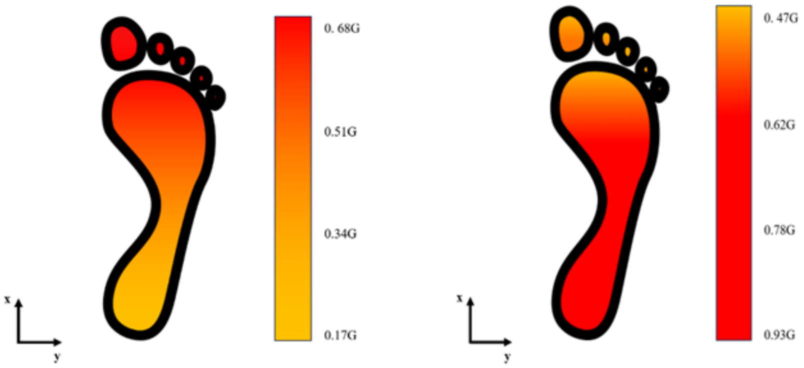

**Figure 1:** Foot force diagrams for Ascending stairs (Left) and Descending stairs (Right)

Next, to estimate the degree of wear in different directions, the model will first determine the specific pressures that the user exerts on the staircase when ascending and descending. According to the references, the specific pressure functions for the upper and lower floors are as follows linear functions:

$$\begin{cases} f_{up}(x) = 1.77mg(a_1 x + b_1) \\ f_{down}(x) = 2.82mg(a_2 x + b_2) \end{cases} \tag{5}$$

where $x$ represents the displacement parallel to the width of the staircase, $f_{up}(x)$ and $f_{down}(x)$ represent the total pressure distribution function when going up and down the stairs, $m$ represents the mass of the user, $g$ represents the acceleration of gravity, whose value is approximately $9.81\,\text{m/s}^2$, and both $a_1, a_2$ and $b_1, b_2$ are constants.

Since the theoretical model for the forces involved in ascending and descending the stairs has been established, it is now possible to, based on the dataset, assign values to the constants: $a_1 = \frac{1}{90000}\text{mm}$ 、 $a_2 = \frac{1}{360000}\text{mm}$ and $b_1 = \frac{1}{600}\text{mm}$ 、 $b_2 = \frac{7}{2400}\text{mm}$ . These values will help refine the model and improve its accuracy in representing the real-world pressure distribution during stair usage.

Now that $f_1(x)$ and $f_2(x)$ has been defined, the model can proceed to refine Archard's wear model in Section 4.2. By incorporating the pressure distributions from both ascent and descent, the wear function regarding moving directions can be updated as follows:

$$\begin{cases} M_{up}(x) = \dfrac{\mu \cdot f_{up}(x) \cdot d_{up} \cdot n_{up}}{H} \\ M_{down}(x) = \dfrac{\mu \cdot f_{down}(x) \cdot d_{down} \cdot n_{down}}{H} \\ M(x) = M_{up} + M_{down} \end{cases} \tag{6}$$

where $M_{up}(x)$、 $M_{down}(x)$ and $M(x)$ represent the degree of wear of the staircase during upward, downward, and overall movement, respectively. The parameters $d_{up}$ and $d_{down}$ are the displacements caused by sliding friction during upward and downward travel, respectively. Based on the available data, the model sets that $d_{up} = 3\,\mathrm{mm}$ and $d_{down} = 5\,\mathrm{mm}$ . Associating the expressions for $M(x)$ in equation (2) and (7) , the following equation can be obtained as follows:

$$\frac{k\ \cdot H}{\mu} = 1.77 \times 10^{-2} mg \cdot d_{up} \cdot n_{up} + 2.82 \times 10^{-2} mg \cdot d_{down} \cdot n_{down} \tag{7}$$

where $k$ denotes the critical value of the slope of the degree of wear, its value can be determined by the upward and downward frequencies $n_{up}$ and $n_{down}$

As the user moves in a straight line up the stairs, The user's path on the stairs can be approximated as a straight-line segment running from the bottom to the top of the stairs. These segments are treated as line elements.

According to Assumption 1, these line elements will pass through several small blocks, each containing discrete wear points that reflect the forces applied. To derive the degree of wear associated with this line element, the model can calculate the slope of the line element by linearly fitting these points. This will enable us to establish an expression for the degree of wear concerning the long distance parallel to the stairs, which should take the following form.

$$k(y) = \frac{\partial M(x,y)}{\partial x} \tag{8}$$

where $x$ represents the displacement parallel to the width of the stair, $y$ represents the displacement parallel to the length of the stair, and $k$ denotes the slope of the line element.

For each stair, based on the slope of the line fitted to the points along that stair, the slopes of the partial line elements must satisfy the following expressions:

$$k(y_i) = \frac{M|_{y=y_i}}{\Delta x} \tag{9}$$

where $i$ is a natural number between 0 and 30, $y_i$ denotes the vertical coordinate of point $i$ , and $\Delta x$ denotes very small increments of $x$ . Since it has $k(y_i)$ , then the model can compute and estimate the average slope of the entire line element

$$\bar{k}(y) = \frac{\sum k(y_i)}{N} \tag{10}$$

where $\bar{k}$ denotes the average slope of the line element, and $N$ is the number of $k(y_i)$ on each stair. Combining the equations (8) and (10) , it gets the expression for $k$ with $y$ as the independent variable. Macroscopically, this equation expresses the relationship between the slope of the degree of wear and the direction of travel.

To determine the user's propensity to ascend or descend the stairs, the model needs to establish a critical value $k_0$, which can be found by setting $n_{up} = n_{down}$ and substituting this into equation (7), i.e.,

$$\begin{cases} k_0 = -0.07 & (\text{Material: Granite}) \\ k_0 = -0.16 & (\text{Material: Oak}) \end{cases} \tag{11}$$

Accordingly, a rubric has been established to determine upward versus downward movement from the extent of wear.

In fact, the model can get a more accurate conclusion by analyzing the situation more scientifically and accounting for potential sources of error, i.e.:

For $k \geqslant 1.1k_0$, consider the user to be moving upward on the stairs.

For $k \leqslant 0.9k_0$, consider the user to be moving downward on the stairs.

For $0.9k_0 < k < 1.1k_0$, consider that the wear produced by the user is independent of the direction of travel.

Under this model, ascent–descent tendencies are inferred from the degree and distribution of wear; the procedure is non-destructive to the staircase.

## 3. Results

### 3.1. Estimations of Use Frequency

The large data prediction model for the user's electricity consumption is implemented in the Clementine software. To make the testing of our conclusions more general, the study randomly selected two staircases of known age from the available dataset, with the materials comprising them being granite and oak, respectively. According to investigations, the wear depths of the points on these two steps have been measured, and their distribution is shown in greater detail in Figure 2.

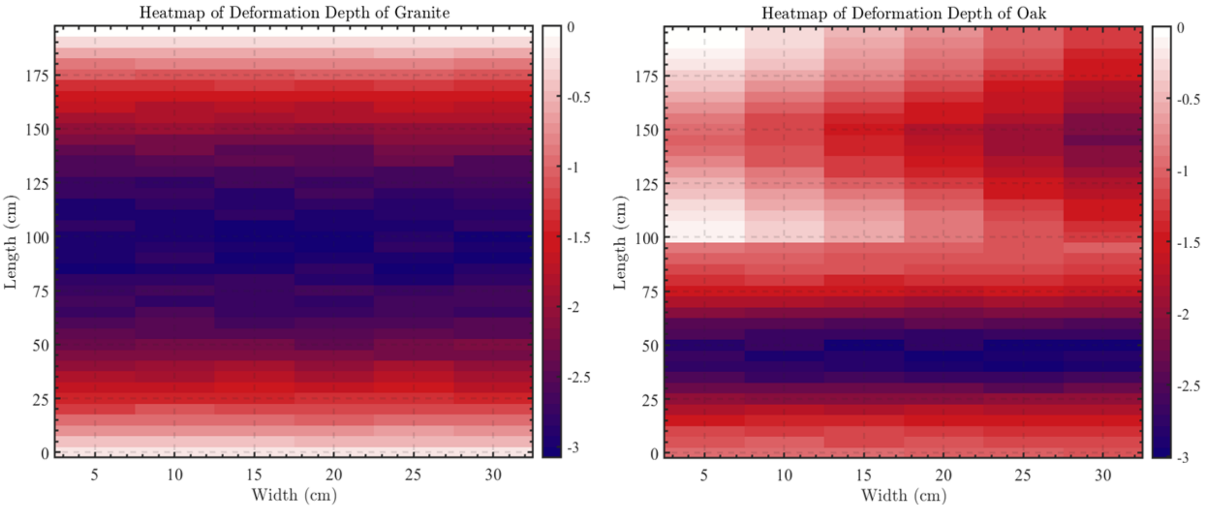

**Table 1:** Depth of Wear: Granite (left) and Oak (right)

By using the model in Section 2.1, the model can measure the average total number of wear instances and the number of treads per day for both staircases, based on equations (3) and (4). The average total number of wear and the number of wear per day for each staircase are obtained as Table 1:

**Table 2:** Stair Wear Modeling Results

| Staircase Materials | Total Number of Wear(per step) | Number of Wear(per day) |
|---|---|---|
| Granite | 42,848,627.181 | 1,173.935 |
| Oak | 3264882.686 | 89.449 |

To further visualize the frequency of stair usage, the data are displayed as a 3D histogram heat map as shown in Figure 3.

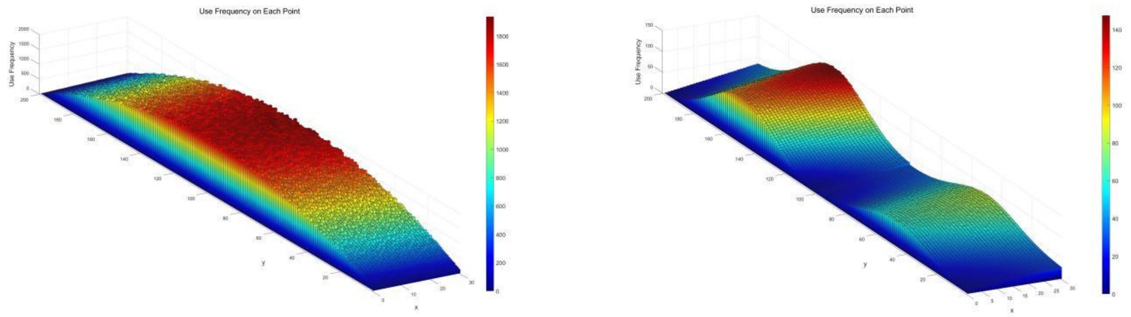

**Figure 2:** Use Frequency on Each Point (Left: Granite; Right: Oak)

By analyzing the frequency of use of each step, it becomes evident that, for granite steps, people tend to walk more frequently through the middle; for oak steps, it is common for people to divide the staircase into left and right areas and use both areas.

Upon comparing the actual data, it found that the oak staircase features a handrail in the center, allowing people to cross the staircase from both sides of the handrail, while the granite staircase lacks such a handrail. This observation strongly aligns with the results the model obtained from our model. Therefore, this model can be applied to estimate the acceptable frequency of stair use.

### 3.2. Direction Preference Analysis

Using the model in Section 2.2, the measured wear data are fitted with a straight line and the slope error is analyzed, as shown in Figure 4.

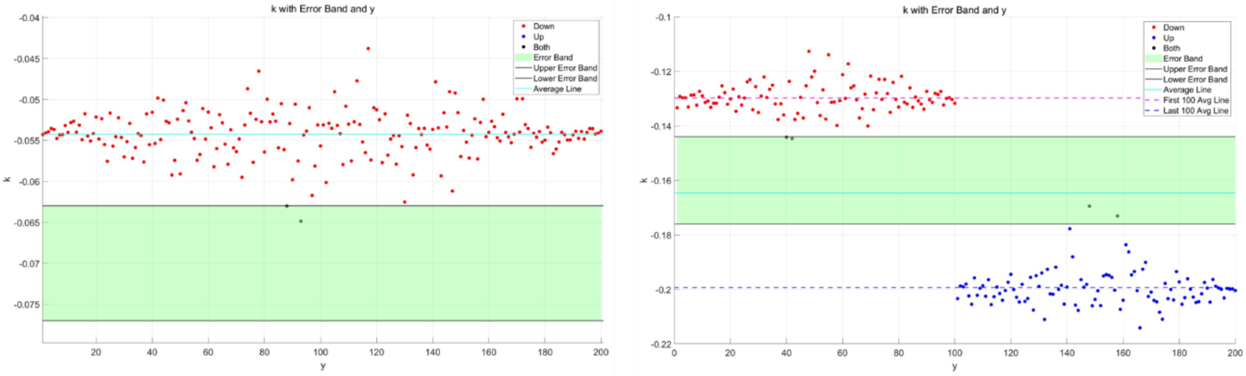

**Figure 3:** with Error Band and $y$ (Left: Granite; Right: Oak)

The centerline of the error band is given by the critical value $k_0$, with an error range of 10% of $k_0$, and the mean of the data points is represented by the mean value $k$. According to this figure, when the mean lies above the error band, users tend to climb the stairs; conversely, when the mean lies below the error band, users tend to descend the stairs. If the mean line lies within the error band, the user does not show a clear preference for upward or downward movement at that point, or the user's behavior on that staircase includes both upward and downward movement. For example, Figure 6 clearly shows that the granite staircase is used predominantly in one direction. In particular, based on the results in Figure 7, the model observe that the staircase made of oak wood shows upward movement in the left zone (0, 100) and downward movement in the right zone (100, 200), which is consistent with the predictions of our model.

## 4. Conclusions

This study establishes an integrated framework for non-destructive inference of staircase use from wear, combining an Optimized Archard formulation for frequency estimation, a direction-sensitive inference scheme, and density-based clustering to delineate same-wear regions. Application to two staircases of known age—granite and oak—demonstrates that material-calibrated wear modeling coupled with spatially resolved depth measurements can recover both aggregate throughput and lateral route choice. The inferred central concentration on granite and bilateral lanes on oak align with observed architectural controls, notably the presence of a central handrail that partitions flows, indicating that the model captures behaviorally meaningful constraints rather than overfitting surface noise. Direction preference analysis using slope-based bands distinguishes ascent- and descent-dominated zones, revealing predominantly unidirectional use on the granite staircase and a bidirectionally segregated pattern on the oak case. Collectively, these outcomes support the model's capacity to translate microtopographic wear into interpretable indicators of circulation intensity, capacity, and preferred movement direction.

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